Calc I
Review for derivatives test
answer Key

① $f(x) = \dfrac{3x^7 - 6\sqrt{x} + 2\sqrt[5]{x^3}}{2x^2}$

$f(x) = \dfrac{3x^7 - 6x^{\frac{1}{2}} + 2x^{3/5}}{2x^2}$

$f(x) = \frac{3}{2}x^5 - 3x^{-\frac{3}{2}} + x^{-\frac{7}{5}}$

$f'(x) = \frac{15}{2}x^4 + \frac{9}{2}x^{-\frac{5}{2}} - \frac{7}{5}x^{-\frac{12}{5}}$

$$\boxed{f'(x) = \frac{15}{2}x^4 + \frac{9}{2\sqrt{x^5}} - \frac{7}{5\sqrt[5]{x^{12}}}}$$

② $g(x) = 5x^9 - 7x^2 + 11$

$$\boxed{g'(x) = 45x^8 - 14x}$$

③ $f(x) = (2x^3 - 5x)(4x - 3) \Rightarrow f(x) = 8x^4 - 6x^3 - 20x^2 + 15x$

$$\boxed{f'(x) = 32x^3 - 18x^2 - 40x + 15}$$

(4) $h(x) = \dfrac{\sin^2(x) + \cos^2(x)}{\csc(x)} \Rightarrow h(x) = \dfrac{1}{\csc(x)}$

$h(x) = \sin(x)$

$\boxed{h'(x) = \cos(x)}$

(5) $f(x) = 3x^5 \cot(x)$

$f'(x) = 15x^4 \cot(x) + 3x^5 \left[-\csc^2(x)\right]$

$\boxed{f'(x) = 15x^4 \cot(x) - 3x^5 \csc^2(x)}$

(6) $g(x) = (x-3)^4 (3x+2)^2$

$g'(x) = 4(x-3)^3 (1) (3x+2)^2 + (x-3)^4 \left(2(3x+2)(3)\right)$

$\boxed{g'(x) = 4(x-3)^3 (3x+2)^2 + 6(x-3)^4 (3x+2)}$

(7) $h(x) = \dfrac{3x^3}{\sin(x)}$

$\boxed{h'(x) = \dfrac{9x^2 \sin(x) - \left[3x^3 (\cos(x))\right]}{\sin^2(x)}}$

7 (other method)

$$h(x) = \frac{3x^3}{\sin(x)} \Rightarrow h(x) = 3x^3 \csc(x)$$

$$h'(x) = 9x^2 \csc(x) + 3x^3(-\csc(x)\cot(x))$$

$$\boxed{h'(x) = 9x^2 \csc(x) - 3x^3 \csc(x) \cot(x)}$$

8 $f(t) = 3t^2 \tan(4t^3)$

$$f'(t) = 6t \tan(4t^3) + 3t^2(\sec^2(4t^3)(12t))$$

$$\boxed{f'(t) = 6t \tan(4t^3) + 36t^3 \sec^2(4t^3)}$$

9 $g(w) = \sec^4(5w^2) = [\sec(5w^2)]^4$

$$g'(w) = 4[\sec(5w^2)]^3 \cdot [\sec(5w^2)\tan(5w^2)] \cdot [10w]$$

$$\boxed{g'(w) = 40w \sec^4(5w^2) \tan(5w^2)}$$

⑩ $h(n) = \sqrt[3]{4x^2-5} \Rightarrow h(n) = (4x^2-5)^{\frac{1}{3}}$

$h'(n) = \frac{1}{3}(4x^2-5)^{-\frac{2}{3}}(8x)$

$$\boxed{h'(n) = \frac{8x}{3\sqrt[3]{(4x^2-5)^2}}}$$

⑪ $g(x) = \sin(\tan(4x))$

$g'(x) = [\cos(\tan(4x))][\sec^2(4x)][4]$

$$\boxed{g'(x) = 4\sec^2(4x) \cdot \cos(\tan(4x))}$$

⑫ $h(t) = 4x\cos^3(5x^2) \Rightarrow 4x[\cos(5x^2)]^3$

$h'(t) = 4\cos^3(5x^2) + 4x\left[3[\cos(5x^2)]^2(\sin(5x^2))\cdot(10x)\right]$

$$\boxed{h'(t) = 4\cos^3(5x^2) + 120x^2\cos^2(5x^2)\sin(5x^2)}$$

(13) $g(x) = 5x^4 - 7x^3 + 2x - 1$

$g'(x) = 20x^3 - 21x^2 + 2$ ← Now take the derivative of this

$$\boxed{g''(x) = 60x^2 - 42x}$$

(14) $f(x) = \dfrac{3x+7}{(3x^2+2)}$

$$f'(x) = \frac{3(3x^2+2) - 6x(3x+7)}{(3x^2+2)^2} = \frac{9x^2+6-18x^2-42x}{(3x^2+2)^2}$$

$$f'(x) = \frac{-9x^2-42x+6}{(3x^2+2)^2}$$

$$f''(x) = \frac{(-18x-42)(3x^2+2)^2 - [(-9x^2-42x+6)\cdot[2(3x^2+2)(6x)]]}{[(3x^2+2)^2]^2}$$

$$f''(x) = \frac{(-18x-42)(3x^2+2)^2 - [12x(3x^2+2)(-9x^2-42x+6)]}{(3x^2+2)^4}$$

(15) $f(t) = 3t^2 - 5t + 1$ at $t = 2$

$f'(t) = 6t - 5$

$f(2) = 3(2)^2 - 5(2) + 1 = 3 \longrightarrow (2, 3)$

$f'(2) = 6(2) - 5 = 7 \longrightarrow m = 7$

$\boxed{\text{Equation: } (y - 3) = 7(x - 2)}$

(16) $f(x) = 3x \cos^2(2x)$ at $x = 0$

$f'(x) = 3\cos^2(2x) + 3x\left[2\cos(2x)\right]\left[-\sin(2x)\cdot(2)\right]$

$f(0) = 3(0)\cos^2(2(0)) = 0 \longrightarrow \boxed{(0, 0)}$

$f'(0) = 3\cos^2(2(0)) + 3(0)\left[2\cos(2(0))\right]\cdot\left[-\sin(2(0))\cdot(2)\right]$

$3(1) + 3(0)\left[2[1]\right]\cdot\left[-0\cdot 2\right]$

$3 + 0 = 3 \longrightarrow \boxed{m = 3}$

$y - 0 = 3(x - 0)$

$\boxed{y = 3x}$

(17) $\lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

$$\lim_{\Delta x \to 0} \frac{-2(x+\Delta x)^2 + 5(x+\Delta x) - 1 - (-2x^2 + 5x - 1)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{-2(x^2 + 2x(\Delta x) + (\Delta x)^2) + 5x + 5(\Delta x) - 1 + 2x^2 - 5x + 1}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\cancel{-2x^2} - 4x(\Delta x) - 2(\Delta x)^2 + \cancel{5x} + 5(\Delta x) \cancel{-1} \cancel{+2x^2} \cancel{-5x} \cancel{+1}}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{-4x(\Delta x) - 2(\Delta x)^2 + 5(\Delta x)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(-4x - 2(\Delta x) + 5)}{\cancel{\Delta x}}$$

$$\lim_{\Delta x \to 0} (-4x - 2(\Delta x) + 5) = \boxed{-4x + 5}$$